

1 The diagram shows a regular octagon $ABCDEFGH$.

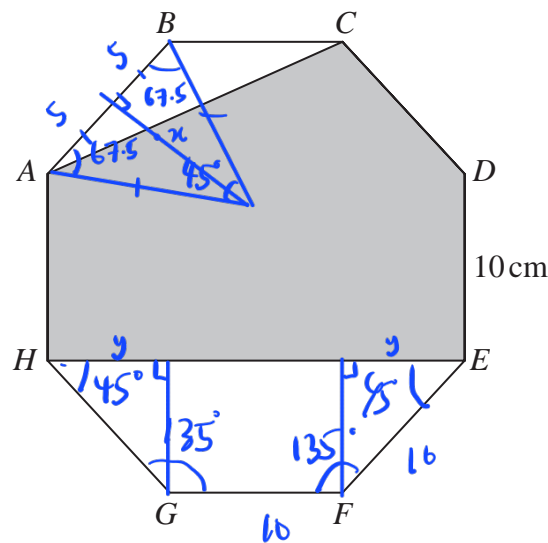


Diagram **NOT** accurately drawn

Each side of the octagon has length 10 cm.

Find the area of the shaded region $ACDEH$.
Give your answer correct to the nearest cm^2

$$\text{Interior angle of octagon} = \frac{(8-2)}{8} \times 180^\circ = 135^\circ \quad (1)$$

split octagon into 8 triangles

$$\text{Find } x: \quad x = 5 \tan 67.5^\circ = 12.07106 \dots$$

$$\text{Area of triangle} = \frac{1}{2} \times 10 \times 12.07106 \dots = 60.355 \dots$$

$$\text{Total area of octagon} = 8 \times 1 \text{ area of triangle}$$

$$\begin{aligned} \text{Area of octagon} &= 8 \times 60.355 \dots \\ &= 482.84 \dots \quad (1) \end{aligned}$$

$$\text{Area of triangle } ABC = \frac{1}{2} \times 10 \times 10 \times \sin 135^\circ = 25\sqrt{2} = 35.355 \dots \quad (1)$$

$$\text{Find } y: \quad y = 10 \cos 45^\circ = 5\sqrt{2}$$

$$\text{Length of HE} = 2 \times 5\sqrt{2} + 10 = 10\sqrt{2} + 10 \quad (1)$$

$$\begin{aligned} \text{Area of trapezium} &: \frac{1}{2} \times (10\sqrt{2} + 10 + 10) \times 10 \sin 45^\circ \\ &= 120.71 \dots \quad (1) \end{aligned}$$

$$\text{Area of shaded region} = \text{Area of octagon} - \text{area of triangle ABC} - \text{Area of trapezium}$$

$$\begin{aligned}\text{Area of shaded region} &= 482.84 \dots - 35.355 \dots - 120.71 \dots \\ &= 326.77 \dots \\ &= 327 \text{ cm}^2 \text{ (nearest cm}^2\text{)}\end{aligned}$$

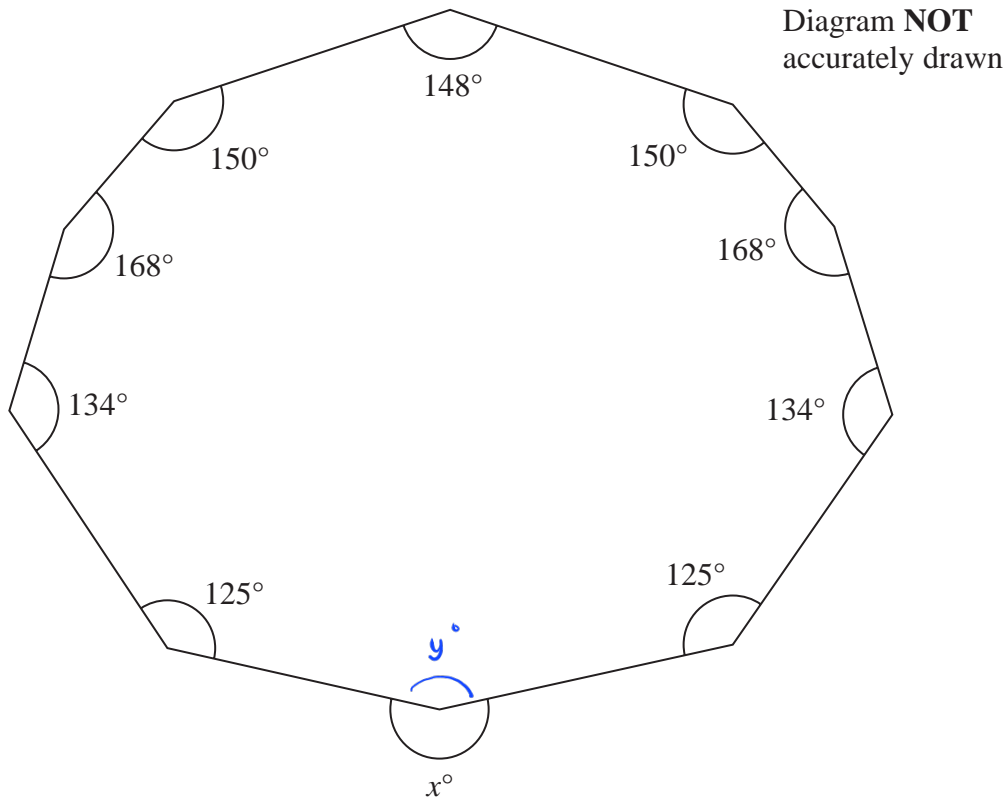
①

327

..... cm²

(Total for Question 1 is 6 marks)

2 Here is a 10-sided polygon.



Work out the value of x .

$$\text{angle inside polygon} : (n-2) \times 180^\circ$$

$$: (10-2) \times 180^\circ = 1440^\circ \text{ (1)}$$

$$125^\circ + 134^\circ + 168^\circ + 150^\circ + 148^\circ + 150^\circ + 168^\circ + 134^\circ + 125^\circ + y^\circ = 1440^\circ$$

$$y^\circ = 1440^\circ - 1302^\circ$$

$$= 138^\circ \text{ (1)}$$

$$\therefore x^\circ = 360^\circ - y^\circ$$

$$: 360^\circ - 138^\circ \text{ (1)}$$

$$: 222^\circ \text{ (1)}$$

$$x = \underline{\underline{222^\circ}}$$

(Total for Question 2 is 4 marks)

- 3 The diagram shows two congruent isosceles triangles and parts of two congruent regular polygons, **X** and **Y**.

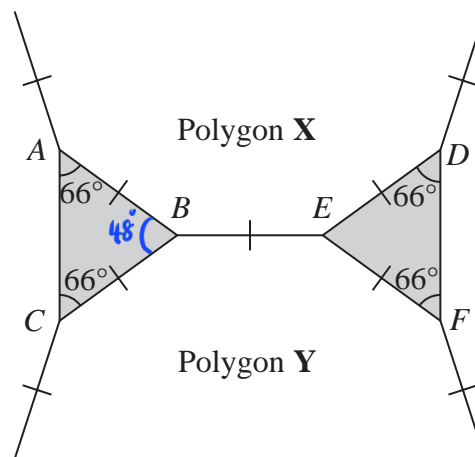


Diagram **NOT**
accurately drawn

The two regular polygons each have n sides.

Work out the value of n .

$$\begin{aligned}\text{angle } ABC &= 180^\circ - 66^\circ - 66^\circ \\ &= 48^\circ \quad \textcircled{1}\end{aligned}$$

$$\begin{aligned}\text{Half of angle } ABC &= \text{exterior angle of polygon X and Y} \\ &= \frac{1}{2} \times 48^\circ = 24^\circ\end{aligned}$$

$$\text{Exterior angle of polygon} = \frac{360^\circ}{\text{no. of sides}}$$

$$24^\circ = \frac{360^\circ}{n}$$

$$n = \frac{360^\circ}{24^\circ} \quad \textcircled{1}$$

$$= 15 \quad \textcircled{1}$$

$$n = \underline{\quad 15 \quad}$$

(Total for Question 3 is 3 marks)

4 The diagram shows parallelogram $EFGH$.

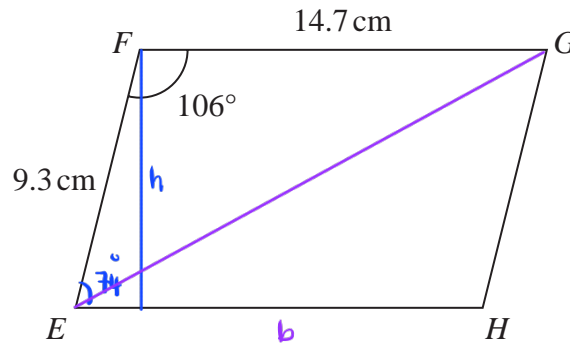


Diagram **NOT**
accurately drawn

$$EF = 9.3 \text{ cm}$$

$$FG = 14.7 \text{ cm}$$

$$\text{Angle } EFG = 106^\circ$$

Area of parallelogram : $b \times h$

(a) Work out the area of the parallelogram.

Give your answer correct to 3 significant figures.

$$\text{angle } FEH = 180^\circ - 106^\circ = 74^\circ$$

$$\sin 74^\circ = \frac{h}{9.3}$$

$$h = 9.3 \sin 74^\circ \quad (1)$$

$$= 8.94 \text{ cm}$$

$$\text{Area of parallelogram} : 8.94 \times 14.7 = 131 \text{ cm}^2 \quad (1)$$

$$\frac{131}{\dots\dots\dots} \text{ cm}^2$$

(2)

(b) Work out the length of the diagonal EG of the parallelogram.

Give your answer correct to 3 significant figures.

By using cosine rule :

$$EG^2 = EF^2 + FG^2 - 2 \times EF \times FG \times \cos 106^\circ$$

$$= 9.3^2 + 14.7^2 - 2(9.3)(14.7) \cos 106^\circ \quad (1)$$

$$= 86.49 + 216.09 + 75.36$$

$$= 377.94 \quad (1)$$

$$EG = \sqrt{377.94}$$

$$= 19.4 \text{ cm} \quad (1)$$

$$\frac{19.4}{\dots\dots\dots} \text{ cm}$$

(3)

(Total for Question 4 is 5 marks)

- 5 The diagram shows a regular pentagon, $ABCDE$, a regular hexagon, $CFGHID$, and a quadrilateral, $EDIJ$.

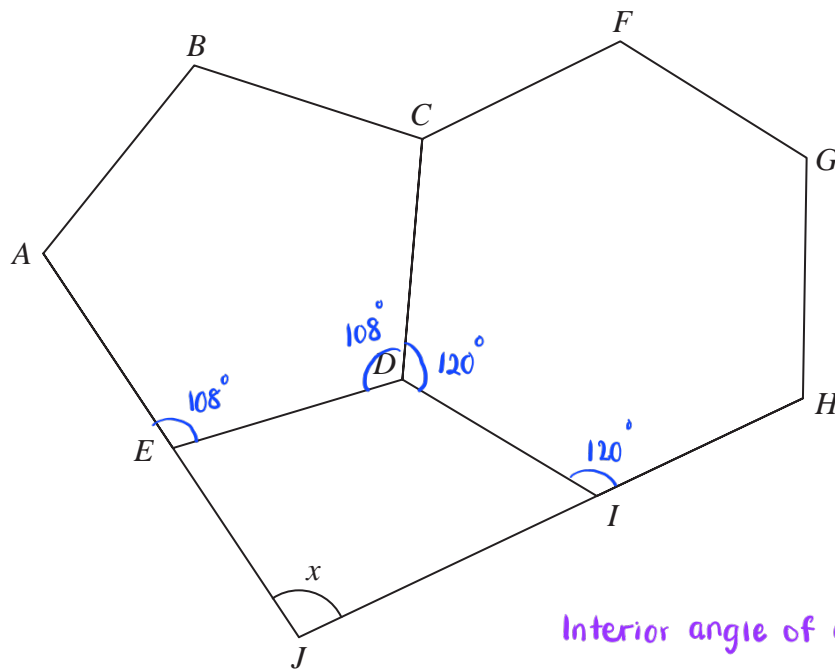


Diagram **NOT**
accurately drawn

AEJ and HIJ are straight lines.

Work out the size of the angle marked x .
Show your working clearly.

Interior angle of a polygon :

$$\frac{n-2}{n} \times 180^\circ$$

where n = number of sides

Finding interior angle of a Pentagon :

$$\frac{5-2}{5} \times 180^\circ = 108^\circ \text{ (1)}$$

Finding interior angle of a hexagon :

$$\frac{6-2}{6} \times 180^\circ = 120^\circ \text{ (1)}$$

$$\text{angle } JED = 180^\circ - 108^\circ = 72^\circ$$

$$\text{angle } EDI = 360^\circ - 108^\circ - 120^\circ = 132^\circ \text{ (1)}$$

$$\text{angle } DIJ = 180^\circ - 120^\circ = 60^\circ$$

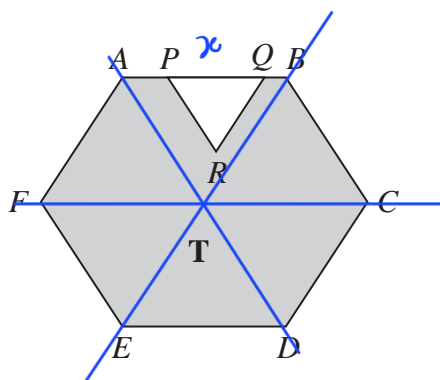
$$x^\circ = 360^\circ - 72^\circ - 132^\circ - 60^\circ \text{ (1)}$$

$$= 96^\circ \text{ (1)}$$

96

(Total for Question 5 is 5 marks)

6

Diagram **NOT**
accurately drawn

The diagram shows a shaded region **T** formed by removing an equilateral triangle PQR from a regular hexagon $ABCDEF$.

The points P and Q lie on AB such that $AB = 1.5 \times PQ$

Given that the area of region **T** is $72\sqrt{3} \text{ cm}^2$

work out the length of PQ .

$$AB = x$$

$$\begin{aligned} \text{Area of one triangle} &= \frac{1}{2} ab \sin C \\ \text{in hexagon} &= \frac{1}{2} x^2 \sin 60^\circ \\ &= \frac{\sqrt{3}}{4} x^2 \quad (1) \end{aligned}$$

$$\begin{aligned} \text{Area of hexagon} &= 6 \times \frac{\sqrt{3} x^2}{4} \\ &= \frac{3\sqrt{3}}{2} x^2 \quad (1) \end{aligned}$$

$$\begin{aligned} \text{Area of } PQR &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \left(\frac{2}{3} x \right)^2 \sin 60^\circ \\ &= \frac{\sqrt{3}}{9} x^2 \end{aligned}$$

$$\text{Area of shaded region} = \left(\frac{3\sqrt{3}}{2} - \frac{\sqrt{3}}{9} \right) x^2$$

$$72\sqrt{3} = \frac{25\sqrt{3}}{18} x^2 \quad (1)$$

$$x^2 = \frac{18 \times 72\sqrt{3}}{25\sqrt{3}}$$

$$= \frac{1296}{25}$$

$$x = \sqrt{\frac{1296}{25}}$$

$$x = \frac{36}{5}$$

$$PQ = \frac{2}{3} AB$$

$$= \frac{2}{3} \times \frac{36}{5}$$

$$= \frac{24}{5}$$

$$= 4.8 \quad (1)$$

4.8

..... cm

(Total for Question 6 is 4 marks)

7 The diagram shows triangle ABP inside the regular hexagon $ABCDEF$

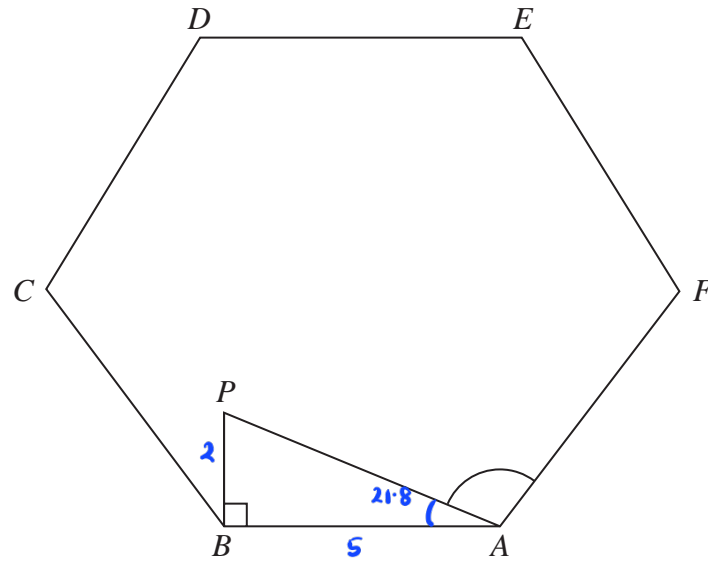


Diagram **NOT**
accurately drawn

$$AB = 5 \text{ cm}$$

$$BP = 2 \text{ cm}$$

$$\text{Angle } ABP = 90^\circ$$

Work out the size of angle PAF

Give your answer correct to 3 significant figures.

$$\begin{aligned} \text{Internal angle of hexagon} &= \frac{6-2}{6} \times 180^\circ \\ &= \frac{4}{6} \times 180^\circ \\ &= 120^\circ \quad (1) \end{aligned}$$

$$\tan BAP = \frac{2}{5} \quad (1)$$

$$\begin{aligned} BAP &= \tan^{-1} \frac{2}{5} \quad (1) \\ &= 21.8^\circ \end{aligned}$$

$$\begin{aligned} \text{angle } PAF &= 120^\circ - 21.8^\circ \quad (1) \\ &= 98.2^\circ \quad (1) \end{aligned}$$

98.2

(Total for Question 7 is 5 marks)

- 8 The diagram shows a regular octagon $ABCDEFGH$ and a regular pentagon $ABIJK$

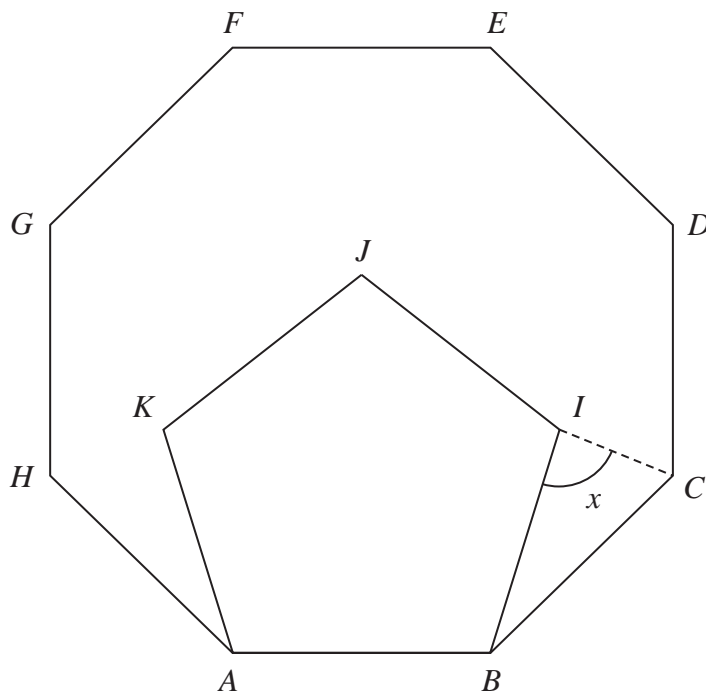


Diagram **NOT**
accurately drawn

Work out the size of the angle x

Interior angle :

$$\text{octagon} : 180^\circ - (360 \div 8) = 135^\circ \quad \textcircled{1}$$

$$\text{pentagon} : 180^\circ - (360 \div 5) = 108^\circ$$

$$\begin{aligned} \angle BCI &= 135^\circ - 108^\circ \quad \textcircled{1} \\ &= 27^\circ \end{aligned}$$

since BCI is isosceles,

$$\begin{aligned} x &= \frac{180^\circ - 27^\circ}{2} \quad \textcircled{1} \\ &= 76.5^\circ \quad \textcircled{1} \end{aligned}$$

76.5

(Total for Question 8 is 4 marks)

9 The diagram shows a regular 10-sided polygon, $ABCDEFGHIJ$

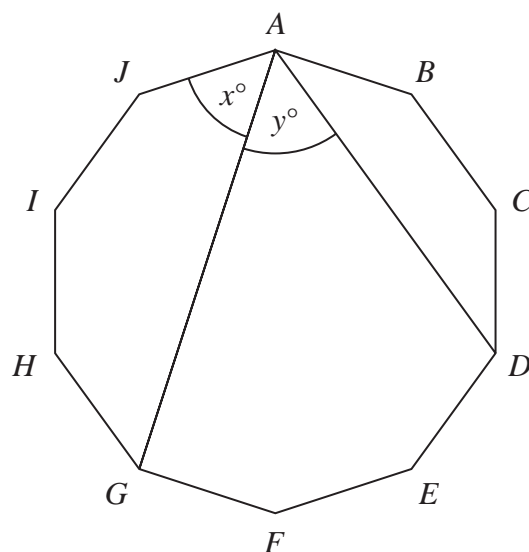


Diagram **NOT**
accurately drawn

Show that $x = y$

$$\text{Interior angle: } \frac{(10-2) \times 180^\circ}{10} = 144^\circ \quad (1)$$

$$x = \frac{540^\circ - 3(144^\circ)}{2} = 54^\circ \quad (1)$$

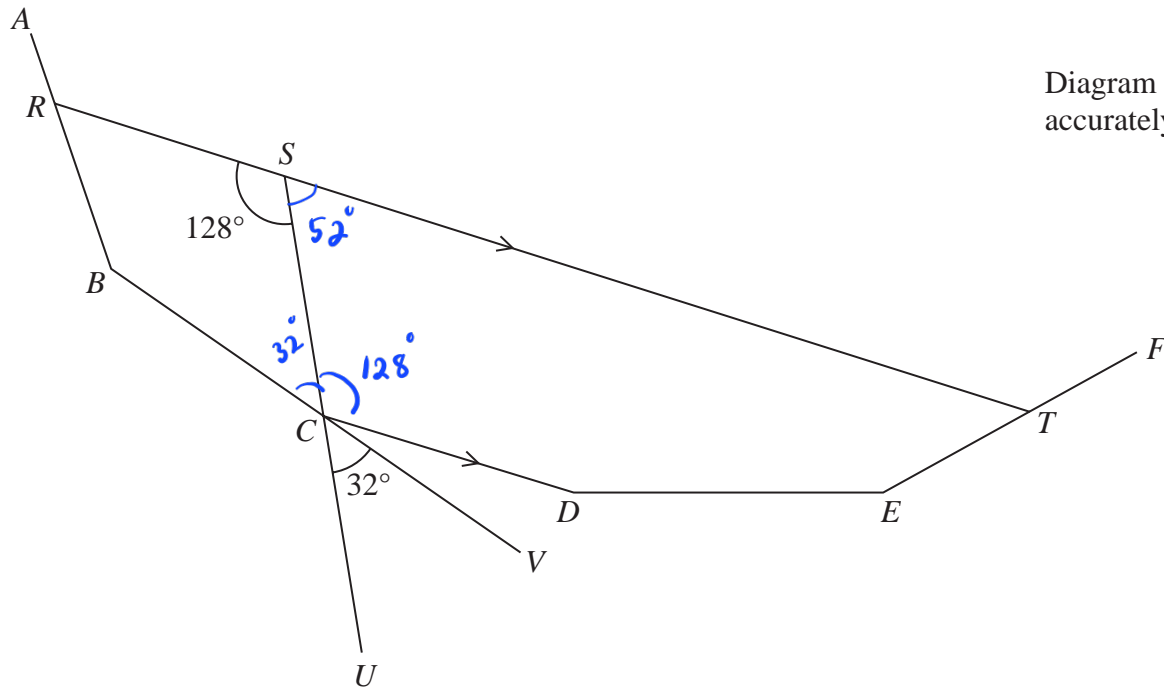
$$\angle BAD = \frac{360^\circ - 2(144^\circ)}{2} = 36^\circ \quad (1)$$

$$\begin{aligned} y &= 90^\circ - 36^\circ \\ &= 54^\circ \quad (1) \end{aligned}$$

$$\therefore y = x$$

(Total for Question 9 is 4 marks)

10

Diagram **NOT**
accurately drawn

AB , BC , CD , DE and EF are five sides of a regular polygon.

RST , SCU and BCV are straight lines.

RST is parallel to CD

Angle $RSC = 128^\circ$

Angle $UCV = 32^\circ$

Work out how many sides the polygon has.

Show your working clearly.

$$BCS = UCV = 32^\circ$$

$$SCD = RSC = 128^\circ \quad (1)$$

$$TSC = 180^\circ - 128^\circ = 52^\circ$$

$$\text{interior angle} = 128^\circ + 32^\circ = 160^\circ \quad (1)$$

$$180(n-2) = 160n \quad (1)$$

$$180n - 360 = 160n$$

$$20n = 360$$

$$n = 18 \quad (1)$$

18

(Total for Question 10 is 4 marks)

- 11 The diagram shows a pentagon.

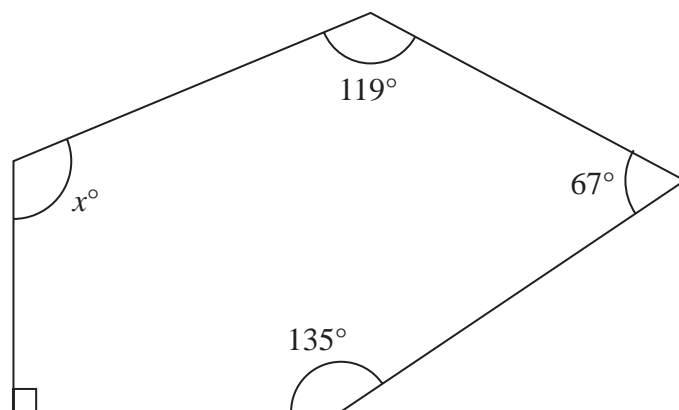


Diagram **NOT**
accurately drawn

Work out the value of x

$$\text{Total angle : } 3 \times 180^\circ = 540^\circ \quad (1)$$

$$540 - 90 - 135 - 67 - 119 \quad (1)$$

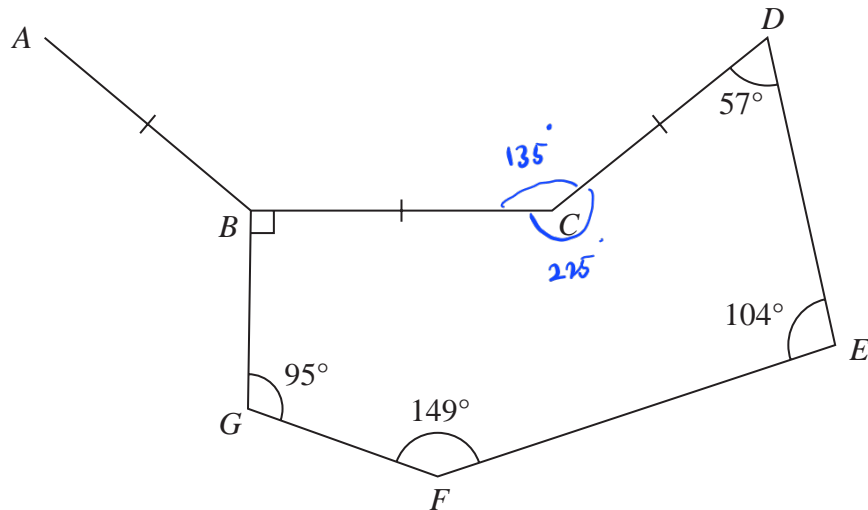
$$= 540 - 411$$

$$= 129 \quad (1)$$

$$x = 129$$

(Total for Question 11 is 3 marks)

12

Diagram **NOT** accurately drawn

$BCDEFG$ is a hexagon.

AB , BC and CD are three sides of a regular n -sided polygon.

Calculate the value of n

Show your working clearly.

sum of
 Interior angle of hexagon : $(6-2) \times 180^\circ = 720^\circ$ (1)

$$\begin{aligned} \text{angle BCD (large)} &= 720^\circ - 90^\circ - 95^\circ - 149^\circ - 104^\circ - 57^\circ \\ &= 225^\circ \quad (1) \end{aligned}$$

$$\begin{aligned} \text{angle BCD (small)} &= 360^\circ - 225^\circ \\ &= 135^\circ \quad (1) \end{aligned}$$

$$\frac{180(n-2)}{n} = 135^\circ$$

$$180n - 360 = 135n$$

$$180n - 135n = 360$$

$$45n = 360$$

$$n = \frac{360}{45} = 8 \quad (1)$$

$$n = \dots\dots\dots 8$$

(Total for Question 12 is 4 marks)

13 The diagram shows two circles with centre O and a regular pentagon $ABCDE$

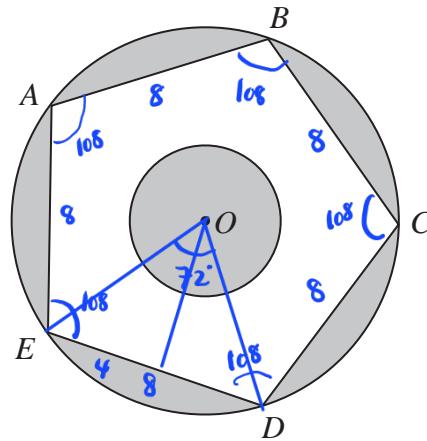


Diagram **NOT**
accurately drawn

A , B , C , D and E are points on the larger circle.
The pentagon has sides of length 8 cm.

The diagram is shaded such that

shaded area = unshaded area

Work out the radius of the smaller circle.
Give your answer correct to 3 significant figures.

$$\text{pentagon angle} = \frac{180 \times 3}{5} = 108^\circ$$

$$\text{angle } EOD = 180 - 54 - 54 = 72^\circ$$

$$\begin{aligned} \text{height of triangle } EOD, \tan 54^\circ &= \frac{\text{height}}{4} \\ &= 4 \tan 54^\circ = 5.505 \dots \quad (1) \end{aligned}$$

$$\frac{\text{length } OE}{\sin 54^\circ} = \frac{8}{\sin 72^\circ}$$

$$OE = \frac{8 \sin 54^\circ}{\sin 72^\circ} = 6.805 \dots = \text{radius of large circle}$$

$$\text{Area of whole diagram} = \pi \times 6.805^2 = 145.489 \dots \quad (1)$$

$$\text{Area of pentagon} = 5 \times \frac{1}{2} \times 8 \times 5.505 \dots = 110.11 \quad (1)$$

shaded area = unshaded area

$$145.489 - 110.11 + \pi r^2 = 110.11 - \pi r^2 \quad (1)$$

$$2\pi r^2 = 74.731... \quad (1)$$

$$r^2 = 11.89...$$

$$r = 3.45 \text{ (3 s.f.)} \quad (1)$$

3.45

..... cm

(Total for Question 13 is 6 marks)

14 Here is a 9-sided regular polygon $ABCDEFGHIJ$, with centre O

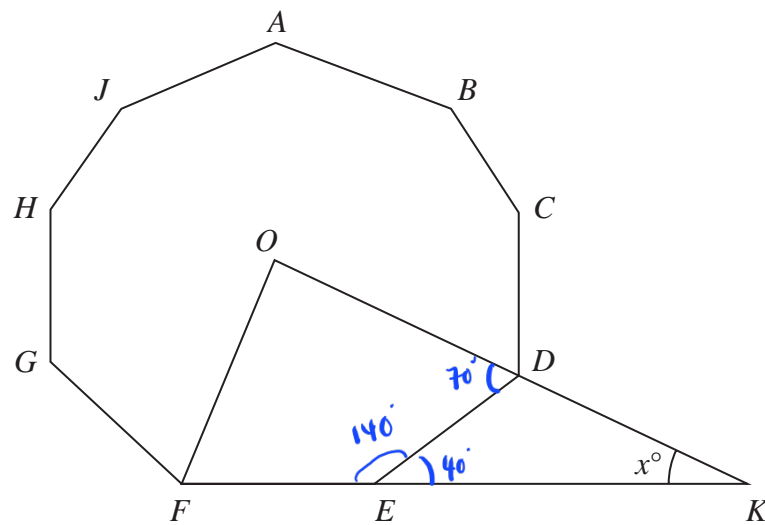


Diagram **NOT**
accurately drawn

ODK and FEK are straight lines.

Work out the value of x

$$\text{interior angle of polygon} = \frac{(9-2)(180)}{9} = 140^\circ \quad (1)$$

$$DEK = 180^\circ - 140^\circ = 40^\circ$$

$$EDK = 180^\circ - \left(\frac{140}{2}\right) = 110^\circ \quad (1)$$

$$x = 180^\circ - 110^\circ - 40^\circ$$

$$= 30^\circ \quad (1)$$

$$x = \underline{\quad 30 \quad}$$

(Total for Question 14 is 3 marks)